

ESTIMATION OF HYDRAULIC CONDUCTIVITY FROM COMPLEX ELECTRICAL MEASUREMENTS

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Abstract

Electrical and hydraulic properties are closely related in porous rock. Complex electrical behaviour is caused by polarization accompanied by charged grain surface and constrictions along the capillaries. Since both electrical phenomena and hydraulic flow are governed by the shape of pore channels, complex electrical conductivity may be used to improve indirect measurement of hydraulic permeability. Determination of the formation factor and surface area is based on a constant phase angle model of conductivity. A modified Kozeny-Carman-equation and the assumption that the internal surface area has a fractal behaviour are both used to estimate permeability. Both experimental and published data are used to study the relationship. Analysis suggests that, in the case of water saturated rock, the estimation of permeability with an accuracy of one order seems to be possible. The model should be applicable for the investigation of relative permeability. It is the purpose of this paper to contribute to the quantification of the effects of permeability on lab measured as well as in-situ complex rock conductivity. These relationships will allow the petrophysicist to better estimate permeability from lab and log as well as surface measurements

Introduction

The transport properties of porous rock and soils, i.e. fluid flow permeability and electrical conductivity, are of great interest in reservoir evaluation but also in near surface environmental applications. Both properties are governed by pore space geometry and related to the microstructure of the pore space. In petrophysics microstructure has to be described by practicable parameters, i.e. formation factor and surface area. The complex nature of electrical rock conductivity is attributed to zones of unequal ionic transport characteristics along the pore channels caused by charged interfaces and constrictions. Complex electrical conductivity has been most extensively studied in rocks without conductive minerals by Vinegar and Waxman (1984), Olhoeft (1985), Börner (1991), Ruffet et al. (1991), Kulenkampff et al. (1993).

Many different models have been developed to relate permeability to electrical parameters (Brace 1977, Wong et al. 1984, Katz and Thompson 1986). Walsh and Brace (1984) used the direct correlation between formation factor F and permeability. The very different exponents found for various rock types are due to significant differences of internal surface area. The Kozeny-Carman-equation, a generalized Hagen-Poiseuille-equation for arbitrary capillary cross sections, relates the permeability of a porous medium with its porosity and other structural parameters (Wyllie and Gardner 1958).

Some publications have shown that complex parameters measured at frequencies below 1 kHz are related to the cation exchange capacity and the size of surface area (Vinegar and Waxman 1984, Börner and Schön 1991). In order to be able to use the relationship of internal surface area and formation factor to electrical conductivity, the Kozeny-Carman-equation with respect to hydraulic permeability will be examined. Thus, permeability can be estimated if water conductivity is known and a fractal behaviour of pore space geometry is assumed (Sen et al. 1981, Pape et al. 1987). Despite the relationship has a more general character, the experimental investigations focus on unconsolidated sands.

CPA-model of low frequency complex electrical conductivity

Laboratory analysis of complex electrical rock conductivity σ^* in a wide frequency range shows a frequency dependence similar to that shown in Figure 1. The shape of the frequency dependence seems to be due to the fractal nature of the pore space geometry (Dissado and Hill 1988).

The phase angle φ is defined as the ratio between imaginary component σ'' and real component σ' according to

$$\tan \varphi = \frac{\sigma''(\omega)}{\sigma'(\omega)} \quad (1)$$

with

$$\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega) \quad (2)$$

whereby φ is practically independent on frequency below 1 kHz. The conductivity spectra can be fitted with a constant phase angle (CPA) model for conductivity σ^* (Jonscher 1981):

$$\sigma^*(\omega) = \sigma_n (i\omega_n)^{1-p} \quad (3)$$

where σ_n is an amplitude factor, ω_n the normalized angular frequency, $1-p$ the exponent describing the power law dependence of conductivity on frequency and $i = \sqrt{-1}$. The exponent $1-p$ is in the order of 0 to 0.03 for silicate rocks. Above 1 kHz the conductivity spectra is commonly fitted with the empirical Cole-Cole-equation (Kulenkampff et al. 1993). This investigation is focused on the frequency range between some millihertz and 1 kHz. In this frequency range the complex conductivity can be explained by a simple electrical parallel conduction of three components (Vinegar and Waxman 1984, Börner 1992):

- real electrolytic conductivity (Archie 1942),
- real interface conductivity and
- imaginary component caused by charge polarization.

The imaginary component σ'' in (2) is a measure of the cation exchange capacity-to-porosity ratio Q_V (Vinegar and Waxman 1984). Q_V is related to the surface area-to-porosity ratio S_v (Patchett 1975). Thus, a correlation is also found between S_v and the imaginary component σ'' as well as the exponent $1-p$ in (3) (Börner and Schön 1991, Kulenkampff et al. 1993). For the investigated sands the relationship between S_v and the imaginary component is plotted in figure 2. The correlation is less significant than in the case of hard rocks. This is due to experimental errors caused by low surface area and structural instability.

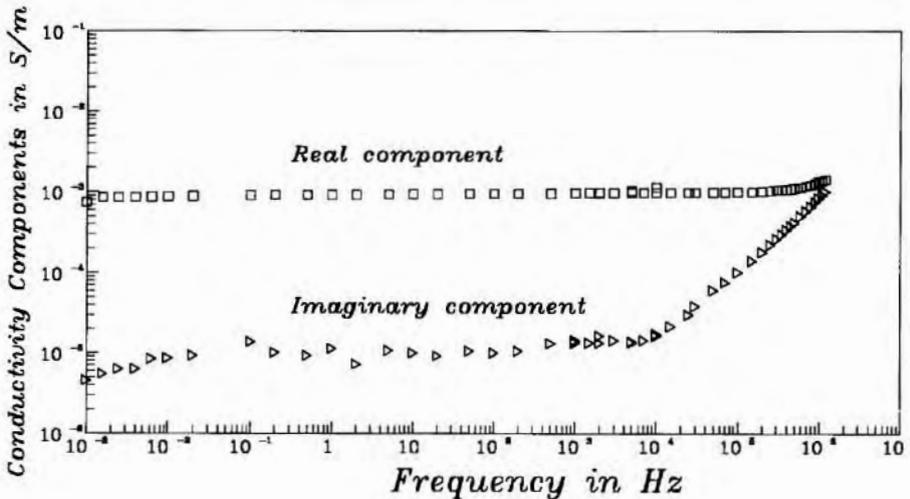


Fig. 1
Frequency dependence of conductivity components (clean sand), Kulenkampff et al. 1993).

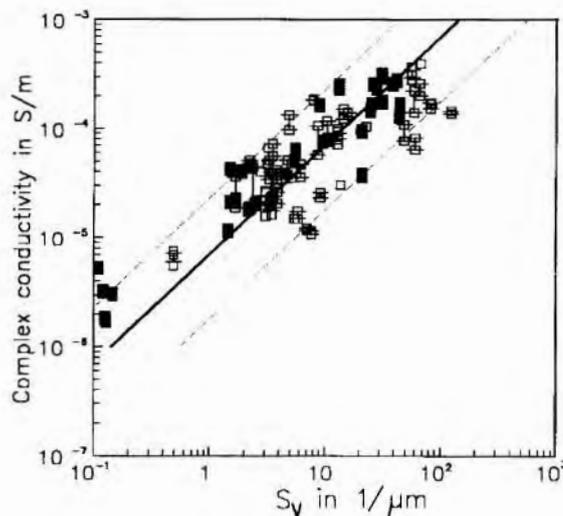


Figure 2
Complex conductivity (imaginary part) vs. surface area-to porosity ratio.
□ unconsolidated pleistocene sediments;
■ sandstones

Permeability and complex electrical conductivity

The permeability k is defined by Darcy's law, which relates fluid flow velocity v to the pressure gradient across the sample length $gradp$:

$$v = \frac{k}{\eta} \text{gradp} = \frac{K}{dg} \text{gradp} \quad (4)$$

η is the dynamic viscosity, d the fluid density and K the permeability coefficient (Darcy velocity). In fresh water and at 20°C $K[\text{m/s}] \approx 1\text{E-}04 k[\mu\text{m}^2]$. The Kozeny-Carman equation relates the permeability of a porous medium with its porosity Φ , tortuosity T and hydraulic radius m :

$$k = \frac{\Phi m^2}{c T} \quad (5)$$

Sen et al. (1981) analysed the relationship between permeability, surface area-to-porosity ratio and porosity using a self similar sandstone model. Pape et al. (1981, 1987) showed that the effective hydraulic radius m can be calculated from surface area-to-porosity ratio for a given fractal dimension of the porous rock. Porosity and tortuosity in equation (5) can be replaced by the true formation factor F . This is used to transform "electrical" pore space parameters into the hydraulic properties. Therefore, the use of equation (5) for calculating permeability from an electrical determined formation factor F and surface area-to-porosity ratio S_V^{el} seems to be possible in the general form:

$$k = a_1 \frac{1}{F(S_V^{el})^\beta} \quad (6)$$

The model discussed here is based on F and S_V determined from one single complex electrical measurement. The separation of the electrolytic bulk conductivity for any brine conductivity gives the true formation factor from Archie equation if brine conductivity σ_w is known. The surface area-to-porosity ratio S_V can be expressed as a function of the low frequency imaginary component (Börner and Schön 1991):

$$S_V^{el} = a_2 \sigma_n'' F \quad (7)$$

σ_n'' is the imaginary component and a_2 is a constant, depending on water composition. In the case of constant fluid conductivities and slightly varying porosities, it is more suitable to use a direct correlation between permeability and electrical parameters:

$$k = a_3 \frac{1}{F(\sigma_n'' F)^\beta} \approx a_3' \frac{1}{F(\sigma_n'')^{\beta'}} \quad (8)$$

Another aspect is to estimate the relative permeability from complex electrical measurements. After replacing the water phase in the pore space by an immiscible fluid (oil, air) the water permeability decreases dramatically. Busch et al. (1993) proposed a sophisticated equation which relates relative permeability $k(S_W)$ to permeability of the water saturated rock and water saturation S_W :

$$k(S_w) / k(S_{w=1}) = S_w^{0.5} \left[1 - (1 - S_w^{1/\bar{m}})^{\bar{m}} \right]^2 \quad (9)$$

(9) is based on a capillary model which considers the distribution of pore sizes (Mualem 1976). Ignoring the deviations from a power law dependence at high values of S_w the following simple relationship can be used:

$$k(S_w) / k(S_{w=1}) = S_w^{\bar{n}} \quad (10)$$

The parameter \bar{m} in (9) is in the order of 0.67 (Busch et al. 1993). Then \bar{n} is expected to be in the order of 3.5 to 5.5. Both F and $S_{v,i}^{el}$ in (6) depend on S_w . In the case of a partially saturated rock, a possible relation between $k(S_w)$ and the saturation dependent electrical parameters F_t and $S_{v,i}^{el}$ is

$$k(S_w) = \alpha_1 \frac{1}{F_t (S_{v,i}^{el})^{\beta_t}} \quad (11)$$

F will increase according to Archie-equation (n - saturation exponent):

$$F_t = F S_w^{-n} \quad (12)$$

The counter ion density in the remaining water phase increases with decreasing water saturation (Vinegar and Waxman 1984). Therefore,

$$S_{v,i}^{el} = S_{v,i}^{el} S_w^{-q} \quad (13)$$

where q is the saturation exponent of the imaginary component. Vinegar and Waxman (1984) found for a group of sandstones $q=n-1$ with q in the order of 1 to 1.3. Börner (1992) obtained q -values in the order of 1.5. Incorporating equations (12) and (13) in (11) an equation similar to (10) is obtained:

$$k(S_w) / k(S_w = 1) = S_w^{n+q\beta_t} \quad (14)$$

It seems (11) may be used as a rough estimate of $k(S_w)$ from complex electrical measurements without an independent S_w -determination. But further experimental investigations are necessary.

Experimental

The relationship between permeability and complex conductivity has been analysed on a series of 40 different unconsolidated rock samples of pleistocene age. The samples were saturated with NaCl-solution. Electrolyte conductivity varies between 0.01 S/m and 0.2 S/m at 25°C. The samples were set up in a cylindrical holder with 30 mm diameter and 80 mm length. Complex conductivities were measured with the laboratory

system shown in figure 3. A multi-electrode cell was integrated into a fluid circulation device (Luckner and Schestakow 1991). Data acquisition and data processing was similar that proposed by Olhoeft (1985). Measurements were carried out at atmospheric pressure and 25 °C. Permeability was determined by water permeation ($\sigma_w=0.1$ S/m, 25°C) and calculated from grain size distribution. Surface area was obtained from BET-nitrogen adsorption measurements.

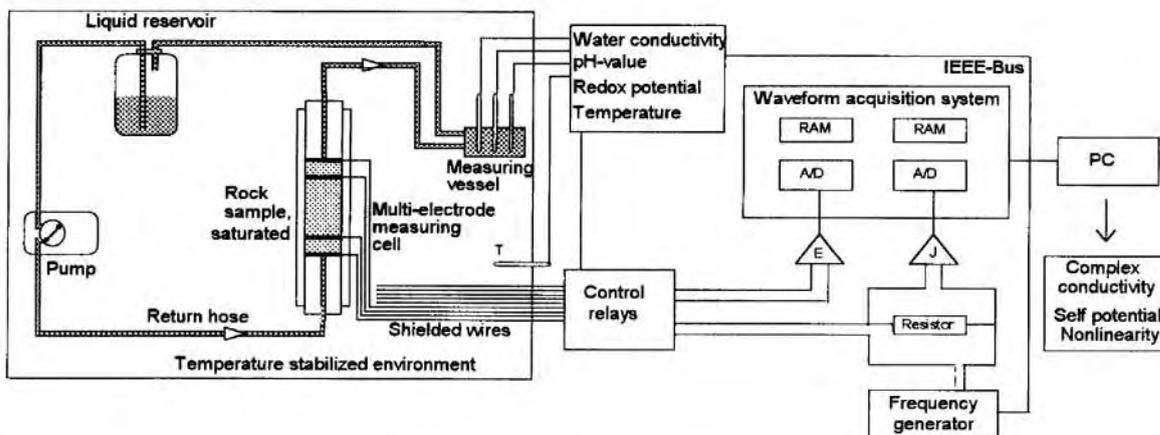


Figure 2 Set up used for low frequency electrical measurements.

Table 1 Petrophysical and electrical properties of some typical unconsolidated sands.

Sample	σ_w [S/m]	σ'_n [S/m]	σ''_n [S/m]	p [-]	Φ [-]	$K(\text{water})$ [m/s]	$K(\text{grain})$ [m/s]	F [-]	S_m [m ² /g]
E60p	0.03	0.00515	1.56e-05	0.998	0.32	0.00467	0.002	6.47	0.62
E67p	0.0289	0.00578	1.98e-05	0.9978	0.31	0.0454	0.002	5.65	0.51
D01a	0.172	0.04429	6.569e-05	0.9989	0.35	3.01e-05	0.00041	4.08	0.65
DS2a	0.1962	0.03989	1.789e-05	0.9997	0.35	0.000581	0.00035	4.99	0.46
DZ7a	0.1566	0.03533	3.942e-05	0.9992	0.36	0.000128	0.00011	4.60	0.82
DZ6a	0.145	0.03528	3.47e-05	0.9993	0.37	8.68e-05	9.3E-05	4.24	1.05
D14a	0.1956	0.04827	3.948e-05	0.9993	0.37	8.8e-06	4.2E-05	4.16	0.69
DZ3a	0.1615	0.03449	5.092e-05	0.999	0.38	2.7e-05	4.0E-05	4.92	0.45
JA2a	0.129	0.02927	7.179e-05	0.998	0.36	0.000169	9.6E-05	4.79	0.71
SX7a	0.1168	0.02165	2.057e-05	0.9992	0.35	0.000657	0.00039	5.57	0.49
SY1a	0.2105	0.04183	3.812e-05	0.9993	0.36	8.82e-05	8.2E-05	5.18	0.90

Results

Water conductivity, complex electrical and petrophysical parameters for some typical sands are given in Table 1. A plot of the product $K \cdot F$ versus σ''_n was made for K determined by flushing the sample with NaCl-solution. The results are shown in figure 4. The plot is presented using data from this investigation and data from Vinegar and Waxman (1984) for shaly sands (0.01 m NaCl-solution). The parameters in equation (8)

were determined by regression $\log(K \cdot F)$ versus $\log(\sigma''_n)$. It was found that the exponent β' is in the order of about 2.8 using K from grain size distribution. Using K from water permeation (figure 4) β' was found in the order of 3.9.

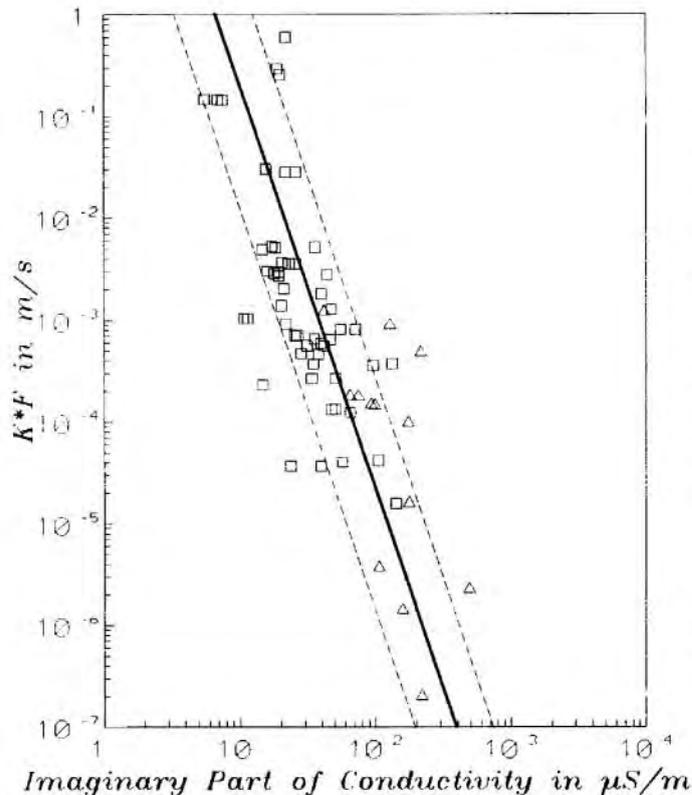


Figure 4

$K \cdot F$ vs. imaginary part of conductivity σ''_n ,
 F and σ''_n from complex electrical measurements,
 K from water flow through measurements.

□ Pleistocene Sand and Gravel, Saxony, Eastern Germany, data from this investigation

△ Shaly Sandstones, different US-Oil fields, data from Vinegar & Waxman (1984)

For the evaluation of pleistocene water aquifers equation (6) was applied on complex conductivity measurements (using a spectral induced polarisation measurement system from Phoenix Ltd.) in field scale (Weller and Börner 1992). The hydraulic conductivities predicted by field measurements were confirmed by independent determination of K from grain size distribution (figure 5). The error bars show the vertical variation of permeability within one aquifer. The electrical data represents an effective permeability for the layer.

In order to obtain some first information with respect to the applicability of equation (11), relative permeability was calculated independently from both conductivity- and S_w -data published by Vinegar and Waxman (1984). The exponent β_t in (11) was varied between 3 and 4 (figure 6). This indicates that the relative permeability predicted from electrical conductivity without knowledge of S_w (equation 9) agrees considerably with $K(S_w)$ calculated independently from water saturation. However, this merely shows self consistency in the choice of relative permeability model because no relative permeability measurements were actually made on the Vinegar and Waxman samples.

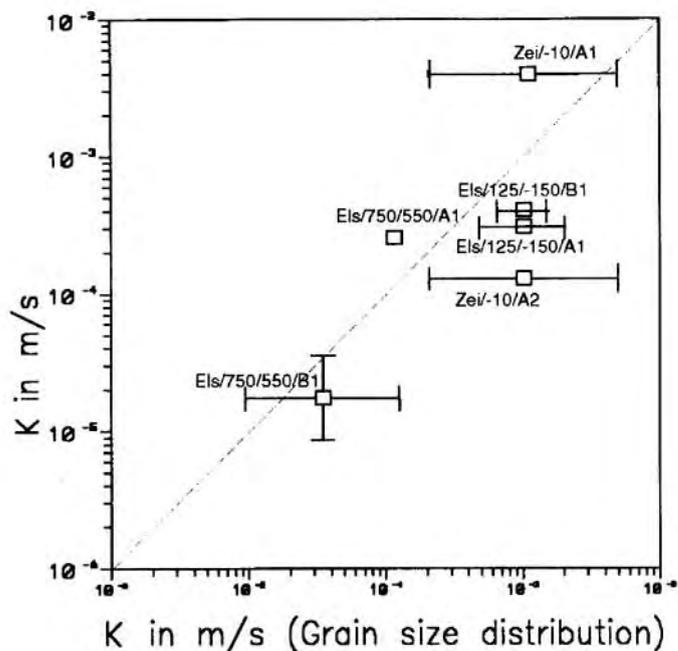


Figure 5
Permeability coefficient estimated from field data (Spectral IP-Measurements) vs. permeability coefficient from grain size parameters.

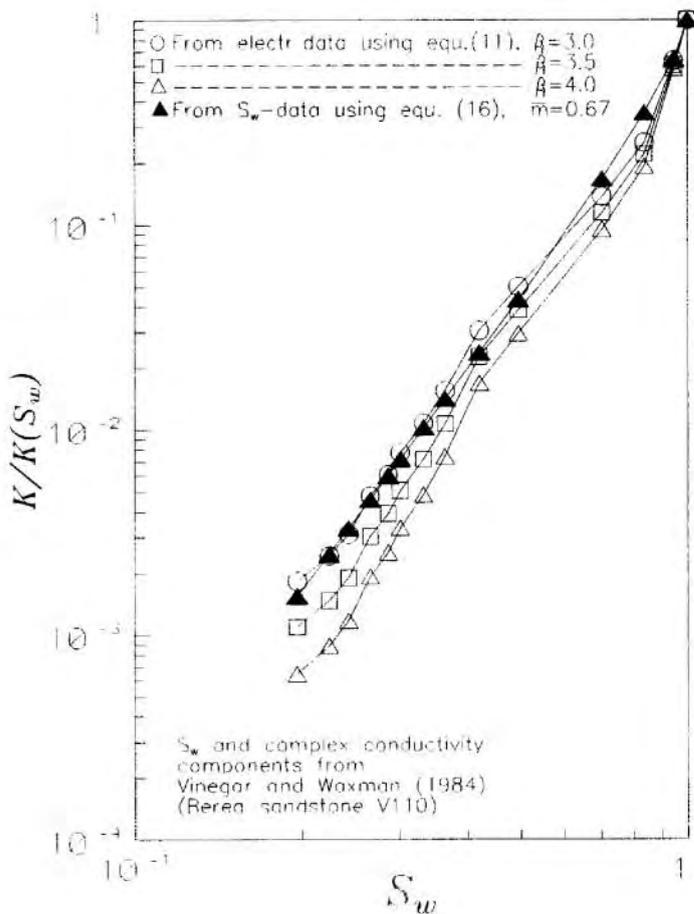


Figure 6
Relative permeabilities, calculated on the basis of complex conductivity data and water saturation data, published by Vinegar and Waxman (1984), sample V110.

□○△ Calculated from complex conductivity data using equation (11);

▲ Calculated from S_w -data using approach (9) from Busch et al. (1993).

Conclusion

A simple electrical conductivity model, based on constant phase angle response in the low frequency region, is applicable to separate electrical bulk and interface effects. Parameters of the pore microstructure are estimated by empirical relations from real and imaginary components. The usage of the formation factor and surface area as determined by one single complex measurement allows to determine, to an order of accuracy, the hydraulic permeability in saturated and unsaturated porous rocks. In case of saturated sands a Kozeny-Carman-like equation has been verified experimentally. For partially saturated sandstones, the relative permeability was calculated using data from Vinegar and Waxman (1984). There was found to exist a good agreement in slope and curve shape with data calculated independently from water saturation.

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