ESTIMATION OF CURVATURE FROM MICRO-CT LIQUID-LIQUID DISPLACEMENT STUDIES WITH PORE SCALE RESOLUTION

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This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Aberdeen, Scotland, UK, 27-30 August 2012

ABSTRACT
Interfacial curvature measurements from computed microtomography (CMT) data collected during 2-phase displacement can potentially provide *in-situ* capillary pressure (P_c) estimates. Results presented by Armstrong et al. (2012) for a simple glass bead pack show favorable agreement between curvature-based and transducer-based capillary pressure measurements, which gives promise for the usage of image-based estimates of P_c in more realistic pore morphologies. However, further investigations are needed to evaluate the currently available curvature measurement methods, in terms of image resolution and quality, such that the limitations are clearly understood. Herein, two approaches are tested to measure interfacial curvature, the first approach measures curvature on triangular surfaces generated at interfacial regions, while the second approach measures curvature directly from an image intensity gradient. A sensitivity analysis was conducted on synthetic and real CMT data to quantify error and then the best approach was applied to multiphase data of a capillary trapped oil blob. In the presented work the limitations and advantages of measuring interfacial curvature from CMT data are addressed and potential approaches for error reduction are highlighted.

1. INTRODUCTION
A subsurface multiphase system consists of two or more immiscible phases and a porous matrix, which results in both solid/fluid and fluid/fluid interfaces. At the fluid/fluid interface there is a discontinuity in pressure called capillary pressure (P_c) and measured as the pressure difference between the nonwetting and wetting phases. In principle, capillary pressure can be measured with a differential pressure transducer that is hydraulically connected to both fluid phases; however, this method is a bulk measurement that disregards local pore-scale pressure gradients and provides no information about the disconnected phase. Recently, a new approach to measure capillary pressure was presented by Armstrong et al. (2012) by measuring fluid/fluid interfacial curvature from CMT data. Herein, we review the various methods available for measuring interfacial curvature from CMT images and test two fundamentally different approaches.

One approach uses explicit geometry, such as an isosurface computed by a marching cube algorithm (*Figure 1b*), then curvature is computed using the averaged polygon normals, or, more accurately, fitted quadric patches (Guggenheimer 1977). With the second approach (*Figure 1c*), curvature is computed directly from the voxel image. Numerous voxel-based approaches exist in the literature; however, some approaches are not suitable for multiphase flow applications. For example, integral mean curvature based on Minkowski functionals lacks the capability to measure curvature on a given interfacial patch (i.e. an interfacial region of interest, such as, a given fluid/fluid interface). Another method counts partial volumes of segmented voxels in a reference sphere centered on each interface voxel (Bullard et al 1995). However, preliminary tests show that this method is very sensitive to the contour pixelization. Also, a differential voxel-based approach can be applied to grayscale data, and thus potentially does not require image segmentation. For example, curvature can be obtained from the second derivatives of the grayscale data and using Gaussian filtering (Monga et al. 1992; Thirion et al. 1995). Lastly, level-set methods presented by Osher et al. 2003 can be used to measure curvature by computing second-order partial derivatives. The mean curvature (k) can be obtained as the divergence of unit normal (N) of the interface (i.e. normalized gradient, *Eq. 1* and *Eq. 2*).

$$\kappa = \nabla . \vec{N} \tag{1}$$

$$\vec{N} = \frac{\nabla \varphi}{|\nabla \varphi|} \tag{2}$$

In *Eq. 2* φ is an implicit function (e.g. gray-scale CMT data or some variation thereof), where an interface is defined by the φ=0 isocontour. Since the second-order derivative is more sensitive to noise than a first-order derivative and moreover initial grayscale image, smoothing such as Gaussian filtering may be required to compensate voxel discretization and related noise, either on initial image or during the calculation (such as, recursive gradient or recursive Laplacian).

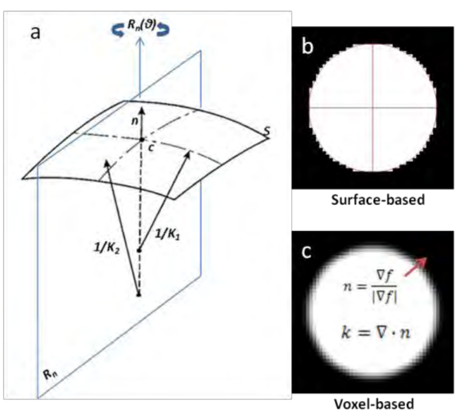

Figure 1: A small segment of an interface (a). The surface is uniquely defined at point C on surface S by the two principals of curvature k_1 and k_2. With the surface-based approach (b), a triangulated surface (red line) is generated from a marching cube algorithm and then curvature is measured locally on the generated surface. With the voxel-based approach (c) the image gradient is used to calculate curvature.

In the multiphase flow community interfacial curvature has mostly been measured from 2D micromodel images using level-set methods (voxel-based approach). Additionally, most of the interest is on mean curvature since the Young-Laplace equation requires this parameter. However, the principal curvatures and/or Gaussian curvature may still be interesting to inquire interface geometry details, to account for parabolic vs. hyperbolic curvatures, and to detect necks and saddle points. Measuring mean curvature, (Cheng et al. 2004) showed experimentally that the nonwetting phase interfacial curvature distribution can be divided into two subsets (1) capillary dominated (i.e. the terminal menisci) and (2) disjoining pressure dominated (i.e. the fluid/fluid interface in the proximity of a solid surface). Also in 2D micromodels, Pyrak-Nolte et al. (2008) used curvature-based capillary pressure measurements to study the relationship between P_c and changes in interfacial area per volume, in terms of wetting phase saturation during imbibition and drainage experiments. Interfacial relaxation, in a smooth-walled divergent channel, was studied by Liu et al. (2011) by measuring interfacial curvature from confocal laser scanning microscopy images by fitting the interfaces to the equation of a circle. As evident in the previously mentioned studies, various pore-scale processes are observable by measuring interfacial curvature. In this report, our intent is to explain the difficulties with measuring curvature, such that, a curvature-based approach for the study of multiphase systems is readily available for further discoveries in more realistic 3D systems.

2. MATERIALS AND METHODS

Three datasets were analyzed: (1) synthetically generated spherical data, (2) CMT generated spherical data, and (3) CMT generated multiphase data. The synthetic data was created by mapping the equation of a sphere into a 50x50x50 pixel space. Sphere diameters analyzed ranged from 5 to 45 pixels. The CMT data was generated by imaging a glass column packed with precision glass spheres (dia = 0.8 mm, resolution = 4.5 μm/pixel) with synchrotron-based CMT (details are presented in Porter et al. 2010). The multiphase data was generated by imaging a sintered glass sample (borosilicate, ϕ = 0.32, K = 20-24 D; Georgiadis et al. 2011) with a Procon x-ray CT-Alpha scanner at 4.1 μm/pixel resolution after water flooding a decane saturated core.

The CMT data was processed using a watershed-based segmentation approach. The following steps were taken: (1) an anisotropic diffusion filter was applied, (2) known regions (i.e. regions internal to a given phase) were marked with a simple threshold, (3) the image gradient magnitude was calculated, and (4) a watershed algorithm was applied. The watershed algorithm utilizes the gradient magnitude to dilate the regions marked in step (2). These regions are dilated, such that, individual phases converge at the inflection point of the gradient magnitude which should correspond to the “optimal” interface between adjacent phases.

With the surface-based approach a marching cube algorithm is applied to the segmented image to generate a triangulated surface. The generalized marching cube algorithm used in Avizo allows for generating interfaces between multiple segmented phases, and can use sub-voxel weights for smoothing surfaces which are critical steps for capturing phase interfaces. After surface generation, curvature is measured by approximating the triangulated surface locally by a quadric form. The eigenvalues and eigenvectors of the quadric form correspond to the principal curvature values and to the directions of principal curvature (e.g. Guggenheimer 1997). This method provides the principals of curvature, the tangent vector, the mean curvature, and the Gaussian curvature for any triangular element on a given surface. The accuracy of this method is controlled by the size of the triangular neighbourhood considered when calculating the best fit quadric form, and can be further smoothed by averaging neighbourhood curvature values. However, more importantly for an accurate curvature calculation is the surface mesh itself. Prior to measuring curvature different levels of smoothing can be applied on the triangulated surface. Surface smoothing occurs by shifting the vertex of each surface element to the average position of its neighbours. Specifically, in Avizo the amount of smoothing is controlled by the number of iterations and the shifting coefficient (ranges from 0 to 1). The shifting coefficient used was 0.4 while the number of iterations was varied. Additionally, the resolution must be high enough to allow for an accurate approximation with the quadratic model.

With the voxel-based approach, initially the image gradient is calculated, then the tangential derivative vectors and 2nd order partial-derivatives are calculated to form a Hessian matrix, from which the eigenvalues provide the principal curvatures. An implementation using MATLAB (S. Arridge) was tested in Avizo using the Avizo MATLAB bridge module. A faster implementation using second-order accurate central differences is currently being implemented as a native module using the Avizo programming interface. With this approach curvature was measured either directly from the original gray-scale image (i.e. direct image segmentation is not required) or a smoothed variation thereof. However, some level of segmentation is required to identify the voxels that represent the interfacial region. Herein, a distance map of the segmented image was used to locate voxels that are 1 pixel distance in length from a given interface. Curvature was measured using the gray-scale raw data, gray-scale data de-noised with an anisotropic diffusion filter, and gray-scale de-noised with a non-local means filter. Curvature was also measured using Gaussian smoothed versions of the segmented data. Gaussian smoothing of the segmented data generates an image intensity gradient over the interfacial regions which can be used to measure curvature. The Gaussian smoothing kernels used ranged in size from 3x3x3 to 12x12x12 pixels with a constant sigma of 0.4.

3. RESULTS

The result of the anisotropic diffusion filter when applied to CMT data is displayed in *Figure 2*. The line profiles clearly demonstrate that noise internal to a given phase is reduced while the image intensity gradient at the bead surface is preserved which is critical for accurate segmentation and curvature measurement. The variability in image intensity internal to a phase and the sharpness of the intensity gradient over the interfacial regions potentially gives a measure for data quality.

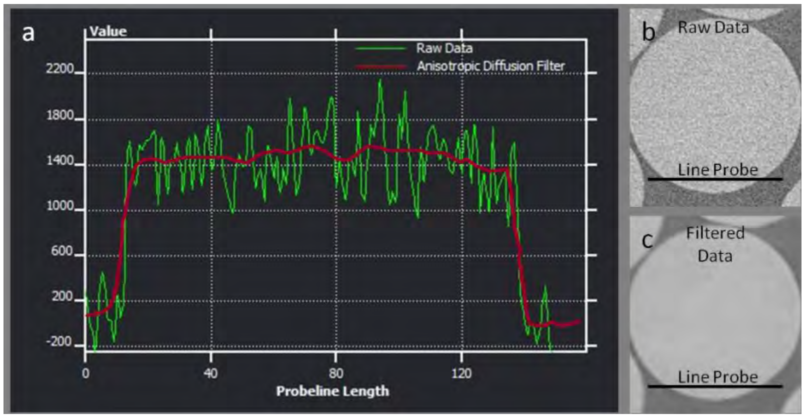

Figure 2: Probe profile (a) for the raw data (b) and anisotropic diffusion filtered data (c).

Curvature results for the ideal sphere data (*Figure 3a*) demonstrate that regardless of the method used or amount of smoothing applied the measured curvature value deviates from the true curvature as the radius of the test sphere decreases. This is basically a pixelation effect due to resolution limitations at the pixelated interface.

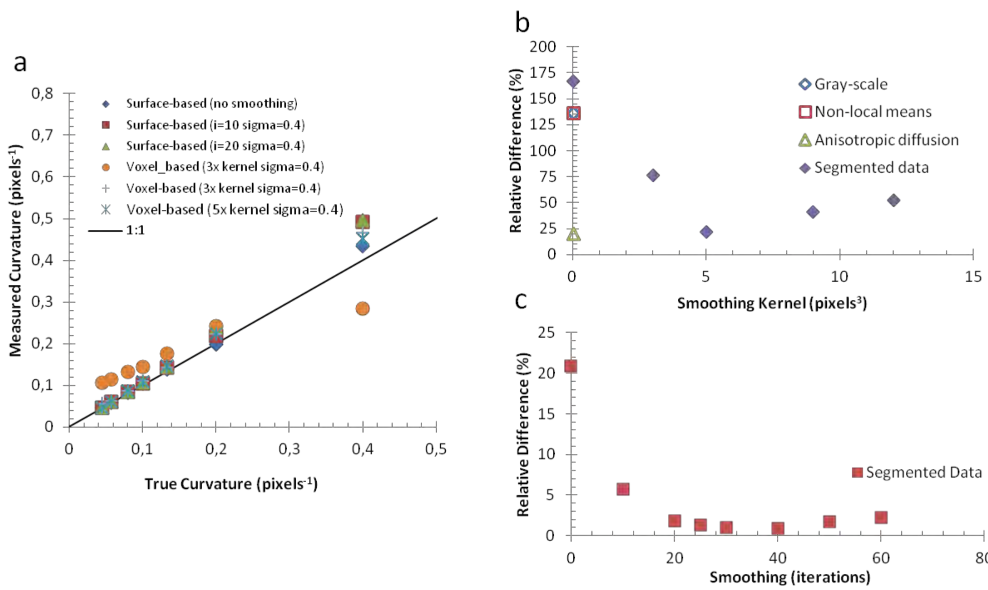

Figure 3: Curvature measured on ideal spheres of various radii (a). Curvature measured on the precision glass bead image using the voxel-based (b) and surface-based (c) approaches.

The precision glass bead data are presented in *Figure 3b* for the voxel-based approach and *Figure 3c* for the surface-based approach. With either approach there is an ideal amount of smoothing where the relative percent difference (RPD) between the measured and true curvature values is minimal. It is noteworthy to mention that when comparing the different voxel-based approaches (*Figure 3b*) the best result was obtained using the anisotropic diffusion filtered gray-scale data. However, overall the surface-based approach gave the best result at 25 smoothing iterations with a ~1% RPD (*Figure 3c*).

Figure 4 represents the type of data obtainable with the coupling of multiphase flow experiments, CMT, and curvature analysis. In *Figure 4* curvature was measured using the surface-based approach with 25 smoothing iterations. The *Figure 4* clearly displays the capillary (yellow/red) and disjoining (blue/green) pressure dominated regions of the residual oil blob interface and could be used to better understand the potential mobility/stability of capillary trapped oil.

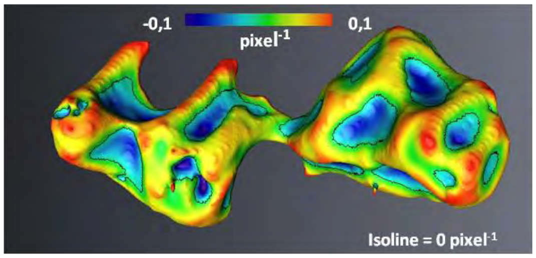

Figure 4: Curvature measured on an isolated oil blob from a water flood experiment imaged with CMT.

4. SUMMARY

Accurate curvature measurement is dependent on image resolution and quality. *Figure 3* is revealing in the respect that image quality is directly related to the amount of smoothing required for optimal accuracy. Further work is required to quantify image quality in the region of interest (i.e. the interfacial regions where the image intensity gradient must be relatively smooth) and then relate quality to the amount of smoothing required to accuracy measure curvature. For instance an autocorrelation function of pixel intensities in the region of interest could provide information about the length scale over which noise is present. However, CMT data is often unique and the optimal settings should be determined on a case-by-case basis.

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