SIMULTANEOUS DETERMINATION OF RELATIVE PERMEABILITY AND CAPILLARY PRESSURE CURVES BY ASSISTED HISTORY MATCHING SEVERAL SCAL EXPERIMENTS

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ABSTRACT
Flooding and centrifuge experiments are conducted using core plugs to determine simultaneously two-phase relative permeability and capillary pressure curves. Traditional interpretation methods of these SCAL (special core analysis) experiments have assumptions which are not always reflecting reality. For example, the unsteady-state experiment assumes that Buckley-Leverett theory is satisfied and therefore the capillary pressure can be ignored.

To avoid these analytical assumptions, computer simulations are used for assisted history matching purposes to interpret SCAL experiments and to simultaneously estimate relative permeability and capillary pressure curves.

The assisted history match approach developed is a combination of two gradient based optimization algorithms: (1) Ensemble Randomized Maximum Likelihood approach (EnRML) and (2) Perturbation Gradient approach (PG). Both have their particular advantages. The EnRML algorithm is able to estimate the uncertainty of the relative permeability and capillary pressure curves. The PG algorithm converges faster than the EnRML algorithm. The combined optimization algorithm is able to combine data from different SCAL experiments (e.g. unsteady state, steady state and centrifuge experiments).

This approach is tested on synthetic cases and a real case. The estimation of the relative permeability and capillary pressure curves improves as more data is used in the history match procedure. Combining unsteady state data with centrifuge data gives a good reconstruction of the true relative permeability and capillary pressure curves.

The main advantage of the assisted history match approach is that the simulation-based interpretation is significantly faster compared to manual history matching. In addition it is more flexible since it can not only be used for regular flooding experiments (i.e. unsteady state, steady state, single-speed centrifuge and multi-speed centrifuge), but also for enhanced oil recovery experiments such as low salinity flooding.

INTRODUCTION
Flooding and centrifuge experiments are conducted using core plugs in order to obtain estimates of two-phase relative permeability and capillary pressure as a function of the saturation of one of the phases. These experiments are referred to as special core analysis
(SCAL) experiments. The classical interpretation of the experiments is by the Johnson-Bossler-Naumann method (JBN) [1] for unsteady state flooding, Darcy [2] for steady state flooding, Hassler-Brunner [3] for multispeed centrifuge and Hagoort [4] for single speed centrifuge with limitations, in the sense that the estimates resulting from the individual interpretations are not mutually consistent. This is because analysis of relative permeability experiments neglects capillary pressure, while analysis of capillary pressure does not take relative permeability effects into account. In other words relative permeability and capillary pressure can be obtained using this classical interpretation; however the curves may not be representative of the fluid flow on a reservoir scale.

To solve this problem, an optimization procedure is proposed that does a “history match” of the experiments estimating all parameters at once. Archer and Wong [5] suggested for the first time to perform a history match on production and pressure data of a core sample. In their study only the relative permeability was varied to achieve a history match. In the last decade many history matching techniques with core flooding experiments are presented in the literature (e.g.: Kerig and Watson [6], Basburg and Karpyn [7], Li et al [8]) to estimate the relative permeability and capillary pressure. The main differences of the history matched interpretation of core experiments are the optimization techniques used and the parameterization techniques to describe the relative permeability and capillary pressure.

This paper presents a new method to interpret the relative permeability and capillary pressure curves from flooding and centrifuge experiments. The new SCAL analysis is described in the method section, specific about the parameterization and optimization methods. In the results section three scenarios are presented. Scenario 1 is a synthetic study, which demonstrates that this method is able to estimate good relative permeability and capillary pressure curves from unsteady state experiments only. Scenario 2 is the simultaneous study, which demonstrates that the simultaneous estimation of the relative permeability and capillary pressure curves of an unsteady state and a centrifuge experiment improves the interpretation. Finally in scenario 3, real core plug measurements are used to demonstrate this SCAL analysis technique.

**METHOD**

The Shell reservoir simulator is coupled with an assisted history matching technique (figure 1) to construct relative permeability and capillary pressure curves. The parameters describing the relative permeability and capillary pressure curves are determined by an optimization technique, which is described in the optimization technique section.

**Parameterization Technique**

In this study the oil and water relative permeability are parameterized by the Corey functions (Brooks and Corey [9]), as seen respectively in equations 1 and 2.

\[
k_{rw}(S_w) = k_{wcor} \left( \frac{1 - S_w - S_{wc}}{1 - S_{wc}} \right)^{n_w} \]  \hspace{1cm} (1)

\[
k_{ow}(S_w) = k_{wcor} \left( \frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{n_w} \]  \hspace{1cm} (2)

In these functions the parameters to be determined are the: connate water saturation ($S_{wc}$), the residual oil saturation ($S_{or}$), the water end point ($K_{wor}$), oil end point ($K_{owe}$), water Corey exponent ($n_w$) and oil Corey exponent ($n_o$).

For the definition of the capillary pressure the Skjæveland parameterization (described by Al-Harrasi et al. [10]) is widely used in literature.

$$p_c(S_w) = A \left( \frac{1}{(S_w - S_{wc})^{\lambda_1}} - \frac{1}{(1 - S_{or} - S_{wc})^{\lambda_2}} \right) + c$$ (3)

Four additional parameters are used to construct the capillary pressure curve are $A$, $\lambda_1$, $\lambda_2$ and $c$, therefore in total 10 parameters are needed to describe the relative permeability and capillary pressure curve. The following constraints must at least be applied to the parameters for this capillary pressure parameterization: $A > 0$, $\lambda_1$, $\lambda_2 > 0$. Also note that the parameter $A$ has a pressure dimension.

For the definition of the capillary pressure the Skjæveland relationship is widely used. However an alternative definition was formulated in this study, because the Skjæveland relationship is able to parameterize the same capillary pressure curve with different set of parameters. The parameter set is therefore not unique, which is difficult for the optimizer to find an optimal solution.

The new parameterization is divided in three saturation zones:

1. The low $S_w$ zone is between the saturation values $S_{wc}$ and $S_{wd}$ where the capillary pressure is defined as:

$$p_c = \frac{c_{wi}}{S_w - S_{wc}} a_{wi} - c_{wi} + S_{wd} \cdot r_i + b_j$$ (4)

2. The mid-range $S_w$ zone, which is between the water-wet and oil wet zone, capillary pressure is linear with the saturation:

$$p_c = S_{wd} \cdot r_i + b_j$$ (5)

3. The high $S_w$ zone is defined between $S_{od}$ and $S_{or}$

$$p_c = \frac{c_{oi}}{1 - S_w - S_{or}} a_{oi} - c_{oi} + S_{od} \cdot r_i + b_j$$ (6)

Six additional parameters are used to describe the capillary pressure curve, which give more control on the capillary pressure curve. In total the relative permeability and capillary pressure can be described by 12 parameters with this new parameterization.
These 12 parameters, which can be estimated by an optimization algorithm, are \( S_{w_c} \), \( S_{o_r} \), \( K_{w_o} \), \( K_{o_w} \), \( n_w \), \( n_o \), \( c_{w_d} \), \( c_{o_l} \), \( r_i \), \( b_i \), \( S_{w_d} \) and \( S_{o_l} \). The parameter \( a_{w_d} \) and \( a_{o_l} \) are fixed and equal to the value of 2 (two).

**Optimization Technique**

For the SCAL experiments, two history matching methods are investigated. The two techniques are the Ensemble Randomized Maximum Likelihood (EnRML) method and Perturbation Gradient (PG) method. The difference between the two techniques is the way the gradient of the objective function with respect to the parameters is estimated, which will be explained later on.

The misfit between predicted and actual values for observations is quantified in the objective function. For every observation, there is a term in the objective function as well as a partial gradient with respect to the parameters to be estimated. Furthermore, every observation has its own (user specified) weight, which reflects the uncertainty in the individual measurements. The objective function itself is identical for both methods, which is defined as:

\[
J = \sum_{i=1}^{N_{obs}} w_i (d_i - y_i)^2
\]  

(7)

Where \( d_i \) is the observation value, \( y_i \) the simulated value and \( w_i \) the weight. The uncertainties in the observations are specified as weights in the objective function, where the weight of an observation is equal to 1/variance. The pressure- and oil production measurements are exponentially distributed, the saturation profile data instead is uniform distributed. The weights used depend on the variance in the measurements and is therefore study dependent. In this study the weights are chosen in the following way: 25 for the oil production, 2500 for the pressure difference and 400 for the saturation profile. These values are based on an estimate of the variance of that particular measurement type. The total number of observations used was limited to a few hundred in total, because adding more measurements would increase the computational time drastically.

**Ensemble Randomized Maximum Likelihood method**

The Ensemble Randomized Maximum Likelihood (EnRML) method [11, 12] calculates gradients of the objective function with respect to the parameters from an ensemble of cases with a specified mean (the nominal case). A case is generated by selecting randomly the parameters from a probability density function. In this study the probability density function was a uniform function with an upper and a lower boundary.

Calculating the gradient is called an iteration, which is repeated until a lower value of the objective function and finally the minimum of the objective function is found. After each individual iteration, the ensemble is resized around its new nominal model using the spread of the parameters in the first iteration. Only the last iteration, the resizing is omitted. The advantage of EnRML is the good approximation of the uncertainty of the solution at the final iteration.
The Perturbation Gradient method

The Perturbation Gradient method provides a finite-difference approximation of the gradient of the objective function with respect to the parameters to be estimated. Cases are generated by perturbing every parameter one-by-one in two directions around the nominal case.

The Perturbation Gradient method does not automatically provide a measure of the uncertainty of the final solution. This uncertainty can theoretically be calculated using the Representer method [13], or a more practical approach can be followed, in which a set of three Ensemble Update iterations is performed after the last Perturbation Gradient iteration.

This approach is referred to as the "combined method". For these last iterations, the ensemble must be resized to its initial spread of the parameters. (In fact, because the updating is a post-processing option, the first Ensemble Iteration will use the Perturbation Gradient cases. The second iteration will be the true Ensemble Update. The last step is required to avoid resizing the final Ensemble results. For this reason, three more iterations are required).

Figure 1: Schematic diagram of the assisted history match approach.

**EXPERIMENTAL SETUP**

**Model Grid**

The core plug is described with a 1-dimensional model grid, with 50 grid blocks. The core plug is modeled vertically, i.e. the number of grid blocks in each horizontal direction is 1. The thicknesses of the first four grid blocks deviate from the other grid blocks: they are 0.1 (outermost blocks), 0.2, 0.4 and 0.8 times the thickness of the block in the center of the model. Furthermore, two grid blocks are added to either side of the model, containing an injector well at one side and a producer well at the other side. The injector well runs on a rate constraint specified by the user. The producer runs on a minimum bottom hole pressure constraint of 1 bar.
**Core Properties**

The SCAL experiment considered in the synthetic scenarios is an unsteady state experiment, with a constant brine injection rate of 0.050 cm³/s during 12 hours. The properties of the core plug and fluid properties are given in Table 1.

Table 1: core and fluid properties for the synthetic case

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core length (cm)</td>
<td>6.0</td>
</tr>
<tr>
<td>Core cross-sect. area (cm²)</td>
<td>11.4</td>
</tr>
<tr>
<td>Absolute permeability (mD)</td>
<td>500</td>
</tr>
<tr>
<td>Porosity (-)</td>
<td>0.3</td>
</tr>
<tr>
<td>Oil density (kg/m³)</td>
<td>730</td>
</tr>
<tr>
<td>Water density (kg/m³)</td>
<td>1000</td>
</tr>
<tr>
<td>Oil viscosity (cP)</td>
<td>1.0</td>
</tr>
<tr>
<td>Water viscosity (cP)</td>
<td>1.0</td>
</tr>
<tr>
<td>Initial water saturation</td>
<td>0.29</td>
</tr>
</tbody>
</table>

**Parameterization Properties**

In Table 2 the values of the parameters describing the permeability and capillary pressure model of the truth model are listed, including the range of the initial guess of the parameters. The connate water and residual oil saturation are in regular analysis measured by a bump flood experiment and fixed on certain value. In this study these values are uncertain and determined by the optimization technique itself. In a flooding experiment the combined effect of low relative permeability and capillary pressure at high water saturations higher than the highest measured pressure in the experiment might seem irrelevant. However, when one of the effects is of lesser importance under the conditions in the reservoir, the residual oil saturation might be significantly overestimated when only results from flooding experiments are taken into account.

Table 2: Values of the parameters used to describe the relative permeability and capillary pressure curves

<table>
<thead>
<tr>
<th>Parameter</th>
<th>“Truth” Value</th>
<th>Initial guess</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relative permeability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Connate water saturation – Swc (-)</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>Residual oil saturation – Sor (-)</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>Water rel. perm. at Sor – kwor (-)</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Oil rel. perm. at Swc – kowc (-)</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Corey exponent oil – no (-)</td>
<td>5</td>
<td>2.00</td>
</tr>
<tr>
<td>Corey exponent water – nw (-)</td>
<td>3</td>
<td>2.00</td>
</tr>
<tr>
<td><strong>Capillary pressure – param. 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water amplitude – cwi (bar)</td>
<td>0.319</td>
<td>0.001</td>
</tr>
<tr>
<td>Oil amplitude – cow (bar)</td>
<td>-0.012</td>
<td>-0.60</td>
</tr>
<tr>
<td>Water domain saturation – Swd (-)</td>
<td>0.400</td>
<td>0.30</td>
</tr>
<tr>
<td>Oil domain saturation – Sod (-)</td>
<td>0.650</td>
<td>0.60</td>
</tr>
<tr>
<td>Linear domain offset bi (bar)</td>
<td>0.0853</td>
<td>-0.15</td>
</tr>
<tr>
<td>Linear domain slope ri (bar/-)</td>
<td>-0.2408</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
RESULTS

Scenario 1: Synthetic unsteady state experiment

The most important advantage of a synthetic case is that the solution (the ‘truth’) is known and therefore can be used for evaluation of the success of a history match. The synthetic measurements used in this scenario are the cumulative oil production, pressure drop and saturation profile in the core. Because these measurements are created by the truth model, noise of 5% is added to these values (see black dots in Figure 2). The measurements are simulated by the reservoir simulator (grey lines in Figure 2) and compared by the optimizer. After an iterative optimization process the output will be an optimal set of parameters, which can explain the measurements.

Figure 2: The measurements used to estimate the relative permeability and capillary pressure curve with the optimizer. Cumulative oil production (left), pressure difference (middle) and saturation profile (right). The truth is represented by the black diamond dots, grey lines represent individual cases; black lines are the nominal (best match) model.

Figure 3: Truth (diamond dots) and initial guess of relative permeability curve (left). Final estimate of relative permeability curve (right). Grey lines represent individual cases; black lines are the nominal (best match) model.

Figure 4: Truth (diamond dots) and initial guess of capillary pressure curve (left). Final estimate of the capillary pressure curve (right). Grey lines represent individual cases; black lines are the nominal (best match) model.
The initial guess of the curves does not match the truth (see left side Figure 3 and 4), the combined optimizer is however able to find a parameterization, which can explain the measurements and is also very close to the truth (see right side Figure 3 and 4). The misfit function (Figure 5) demonstrates that the difference between the measurements and the simulated measurement, with the nominal (best match) model minimal at the last iteration.

There is a small mismatch between the truth and the nominal capillary pressure curve, especially close to the connate water saturation and residual oil saturation. In this case the truth relative permeability and capillary pressure curves (and parameters) are successfully estimated.

![Figure 5](image.png)

Figure 5: Evolution of the (normalized) misfit function for scenario 1, note the increase in the last iterations caused by resizing the parameters to the initial spread. At the last iteration a representative uncertainty is estimated of all parameters.

**Scenario 2: Synthetic combined unsteady state and multiple centrifuge experiment**

In scenario 2 two SCAL experiments are performed to determine the relative permeability and capillary pressure. A synthetic multi-centrifuge experiment and an unsteady state experiment is performed on the same core plug. The properties of the core plug used in this scenario are the same as for scenario 1 (see table 1 and 2).

In order to simulate the centrifuge experiment the centrifugal acceleration is modeled by changing the gravity in the reservoir simulator. The design of the experiment is listed in Table 3.

<table>
<thead>
<tr>
<th>Time</th>
<th>Centrifugal accelerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUR</td>
<td>M/S/S</td>
</tr>
<tr>
<td>0</td>
<td>-741</td>
</tr>
<tr>
<td>4</td>
<td>-1480</td>
</tr>
<tr>
<td>8</td>
<td>-2220</td>
</tr>
<tr>
<td>14</td>
<td>-4000</td>
</tr>
<tr>
<td>38</td>
<td>-6000</td>
</tr>
<tr>
<td>62</td>
<td>-8000</td>
</tr>
<tr>
<td>86</td>
<td>-10000</td>
</tr>
<tr>
<td>110</td>
<td>Time of the last measurement</td>
</tr>
</tbody>
</table>

Table 3: Experimental design of centrifuge experiment
The measurements used in the combined experiment are the oil production of the centrifuge experiment, and the pressure difference and oil production of the unsteady state experiment (Figure 6).

Again 5% measurement noise is added to this synthetic measurements. After the assisted history match a range of simulations are around the measurements (grey lines) and the simulated measurements (red line) are very close to the observed measurements.

Figure 6: The measurements used to estimate the relative permeability and capillary pressure curve with the combined optimizer. Cumulative oil production of the centrifuge experiment (left), pressure difference of unsteady state experiment (middle) and Cumulative oil production of unsteady state experiment (right). The truth is represented by the black dots, grey lines represent individual cases; black lines are the nominal (best match) model.

The assisted history match resulted in estimated values for the parameters and therefore the relative permeability and capillary pressure curve (Figure 7 and 8).

The final estimate of the relative permeability is improved compared to scenario 1, especially the oil relative permeability curve. And the capillary pressure curve is also improved at lower water saturations. This can be explained by the additional measurements from the multi-centrifuge experiment. The centrifuge experiments give only information on the relative permeability of the expelled phase and capillary pressure curve.

Figure 7: Truth (diamond dots) and initial guess of relative permeability curve (left). Final estimate of relative permeability curve (right). Grey lines represent individual cases; black lines are the nominal (best match) model.
Figure 8: Truth (diamond dots) and initial guess of relative permeability curve (left). Final estimate of the capillary pressure curve (right) for the combined method with 50 cases. Grey lines represent individual cases; black lines are the nominal (best match) model.

Scenario 3: Real case

The real case is an unsteady state experiment of an oil wet sample. The measurements used to determine the relative permeability and capillary pressure are oil production measurements and pressure difference measurements (Figure 9). Especially the pressure measurements were difficult to measure, which resulted in a high variance. The optimizer is robust enough to find a optimal solution in the parameter space, even with inaccurate measurements. The best model can explain the measurements very well (black line Figure 9). The corresponding best match curves (Figure 10) are typical oil wet and are very comparable with earlier studies performed on this oil wet sample.

Figure 9: The measurements used to estimate the relative permeability and capillary pressure curve with the optimizer. Cumulative oil production (left) and pressure difference (right). The measurements are represented by the black dots, grey lines represent individual cases; the black line is the nominal (best match) model.

Figure 10: Truth (black dots) and initial guess of capillary pressure curve (left). Final estimate of the capillary pressure curve (right) for the combined optimizer method with 50 cases. Grey lines represent individual cases; black lines are the nominal (best match) model.
CONCLUSIONS
This study shows an assisted history match workflow to estimate relative permeability and capillary pressure curves from SCAL experiments. In the workflow the connate water saturation and residual oil saturation are not assumed known, but are estimated in an assisted history matching process. Especially the residual oil saturation is of interest, because in a flooding experiment, the combined effect of low relative oil permeability and capillary pressure at high water saturations determine the amount of oil that can be recovered. The advantage of this assisted history matching workflow is that the estimated relative permeability and capillary pressure curves are consistent and no assumptions are made as in analytical interpretation methods. The approach is able to estimate the “truth” relative permeability curves and capillary pressure curve. Combining unsteady state data with multi-centrifuge data provides an advantage for the method. The combined algorithm is able to quantify the uncertainty of the permeability curves and capillary pressure curve, which can be used in different scenarios by reservoir simulations. The optimizer method described here is very robust and a fast method for the assisted history match approach performed on SCAL experiments.

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NOMENCLATURE
\( k_{ro} \): Oil relative permeability
\( k_{rw} \): Water relative permeability
\( S_w \): Water saturation
\( S_{wc} \): Connate water saturation
\( S_{or} \): Residual oil saturation
\( K_{ovec} \): Oil end point (at connate water saturation)
\( K_{wor} \): Water end point (at residual oil saturation)
\( n_o \): Oil Corey exponent
\( n_w \): Water Corey exponent
\( P_c \): Capillary pressure curve

\( A \): Pressure amplitude
\( \lambda_1 \): Cap curve water exponent
\( \lambda_2 \): Cap curve oil exponent
\( c \): constant
\( c_{wi} \): Water amplitude
\( c_{oi} \): Oil amplitude
\( S_{wd} \): Water domain saturation
\( S_{od} \): Oil domain saturation
\( b_i \): Linear domain offset
\( r_i \): Linear domain slope
\( d_i \): Observations
\( y_i \): Simulated observations
\( w_i \): Weight
REFERENCES